

Exercise Sheet 8

1. A simple random sample of three observations is taken from a population with mean μ and variance σ^2 . The three sample random variables are denoted Y_1, Y_2, Y_3 . A sample statistic is being sought to estimate μ . The statistics being considered are
 - (a) i. $A_1 = \frac{1}{3}(Y_1 + Y_2 + Y_3)$;
 - ii. $A_2 = \frac{1}{2}(Y_1 + Y_2)$;
 - iii. $A_3 = \frac{1}{2}(Y_1 + Y_2 + Y_3)$;
 - iv. $A_4 = 0.75Y_1 + 0.75Y_2 - 0.5Y_3$.
 - (b) Which of these statistics yield unbiased estimators of μ ?
 - (c) Of those that are unbiased, which is the most efficient?
 - (d) For each unbiased estimator, find the efficiency relative to A_1 .
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2. You are interested in the mean duration of a spell of unemployment for currently unemployed women in a particular city. It is known that the unemployment duration of women is normally distributed with variance 129.6. A random sample of 20 unemployed women yields an average unemployment duration of 14.7 months. Obtain a 98% confidence interval for the population mean unemployment duration for women.
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3. A random sample of 15 pupils attending a certain school is found to have an average IQ of 107.3 with a sample variance of 32.5.
 - (a) Calculate a 95% confidence interval for the unknown population mean IQ, stating any assumptions you need to make.
 - (b) Explain whether you would be happy with a parent's claim that the average IQ at the school is 113.
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4. In an opinion poll based on 100 interviews, 34 people say they are not satisfied with the level of local Council services.
 - (a) Find a 99% confidence interval for the true proportion of people who are not satisfied with local Council services.
 - (b) The Council claims that all but 20% of residents are satisfied with Council services. Use the confidence interval to test this claim, using a 1% level of significance. Explicitly specify the null and alternative hypotheses you use.
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5. Weekly wages in a particular industry are known to be normally distributed with a **standard deviation** of £2.10. An economist claims that the mean weekly income in the industry is £72.40. A random sample of 35 workers yields a mean income of £73.20. Consider the null hypothesis $\mu = 72.4$ with **each** of:

a two-sided alternative hypothesis,
an upper one-sided alternative hypothesis.

- (a) Obtain the p values for the tests (ie, a p -value for each of a two-sided and an upper one-sided alternative hypothesis).
- (b) Comment on the extent of sample support for the null hypothesis in relation to each of the alternative hypotheses.
- (c) Use the p values to perform the corresponding classical hypothesis tests at a 5% level of significance. How do your conclusions relate to those of Question 2, Tutorial Exercise 7?