EC 202

Problem Set 1. Preferences and Utility

1. One of the most commonly used functions in microeconomics is the Cobb-Douglas function

$$y=(x_1)^{\alpha} (x_2)^{\beta}$$

where α , β are positive and <1 parameters.

i. Show that this function is quasi-concave.

ii. Show that if $\alpha + \beta > 1$ than the Cobb-Douglas function is not concave (which shows that not all quasi concave functions are concave).

2. Proof of the Envelop Theorem (this may help to develop some intuitions!!).

Suppose we wish to maximise a function of 2 variables and that the value of this function also depends on a parameter, a : $f(x_1,x_2,a)$. This maximisation problem is subject to a constraint that can be written as: $g(x_1,x_2,a)=0$.

i. **Write down the Lagrangian expression of the FOC for this problem.**

ii. **Sum the two FOCs involving the x s.**

iii. **Now differentiate the above sum with respect to a- this show how the x s must change as a changes while requiring that the FOCs continue to hold.**

iv. **As shown in the notes (LN 1) both the objective function and the constraint can be stated as a function of a: $f(x_1(a),x_2(a),a)$; $g(x_1(a),x_2(a),a)$. Differentiate the first of these with respect to a . This shows how the value of the objective changes as a changes while keeping the x's at their optimal values. You should have terms that involves the x's and a single term in $\partial f/\partial a$.**

v. **Now differentiate the constraint as formulated in part (iv) with respect to a. You should have terms in the x's and a single term in $\partial g/\partial a$.**

vi. Multiply the results from part (v) by λ (the Lagrange multiplier) and use this together with the FOCs from part (iii) to substitute into the derivative from part (iv). You should be able to show that:

$$\frac{df(x_1(a), x_2(a), a)}{da} = \frac{\partial f}{\partial a} + \lambda \frac{\partial g}{\partial a}$$

Which is just the partial derivative of the Lagrangean expression when all the x's are at their optimal values. This proves the theorem. Explain intuitively this result.

3. Showing convexity of indifference curves. Calculation of the MRS for specific utility function is often a good shortcut for showing convexity of ICs and the process can be much easier than applying the definition of quasi concavity (although is much more difficult to generalise to more than 2 goods!!). Show whether or not the following utility functions are represented by convex ICs.

i. U(x,y)=$\sqrt{x \times y}$
ii. U(x,y)=$x + y + xy$
iii. U(x,y)=$\sqrt{x^2 + y^2}$

4. Consider a Cobb Douglas utility function U(x,y)= $x^{\alpha} + y^{\beta}$.

i. Calculate the MRS. Does this result depend on whether $\alpha + \beta = 1$? Does this sum have any relevance to the theory of choice?
ii. For commodity bundles for which x=y, how does the MRS depend on the values of α and β? Give an intuition for why if $\alpha > \beta$, MRS>1. Illustrate your argument with a graph.
iii. Suppose an individual obtains utility only from amounts of x and y that exceed minimal subsistence levels given by x_0 , y_0. In this case

$$U(x,y) = (x - x_0)^{\alpha} + (y - y_0)^{\beta}.$$

Is this function homothetic?

5.

i. Show that the CES function

$$\alpha \frac{x^\delta}{\delta} + \beta \frac{y^\delta}{\delta}$$

is homothetic. How does the MRS depends on the ratio y/x?

ii. Show that the MRS is strictly diminishing for all values of $\delta < 1$.

6. Consider the function U(x,y)= x+ln y. Note that the function is linear in x and non-linear in y, for this reason it is referred as quasi-linear utility function.

i. Find the MRS and interpret the results.
ii. Show that the function is quasi concave.
iii. Find the equation for the IC for this function.
iv. Compare marginal utility of x and y. How do you interpret this function?
v. Consider how the utilities changes as the quantities of the two good increase, describe some situations where this function might be useful.