Inverse of a 2×2 matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Find $\begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}^{-1}$.

Is $\begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix}$ invertible?

Find A^{-1} where $A = \begin{pmatrix} 2 & 7 \\ -1 & 3 \end{pmatrix}$ and solve $A\underline{x} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$

$\boxed{\text{Inverse of a } 2\times 2 \text{ matrix}}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{\underbrace{ad-bc}_{\text{determinant}}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \quad \text{if } ad-bc \neq 0$$

$$X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

adjugate / adjoint

$$\operatorname{adj}(X) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\begin{aligned} \operatorname{adj}(X)\, X &= \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \\ &= \begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix} \\ &= (ad-bc) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

Inverse of a 2×2 matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \quad \text{if } ad-bc \neq 0$$

Find $\begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}^{-1}$.

determinant $= 3\times 4 - 2\times 5$

$= 12 - 10 = 2 \neq 0$

so inverse exists

$$\begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}^{-1} = \frac{1}{2}\begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 2 & -\frac{5}{2} \\ -1 & 3/2 \end{pmatrix}$$

check

$$\begin{pmatrix} 2 & -\frac{5}{2} \\ -1 & 3/2 \end{pmatrix}\begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 6-5 & 10-10 \\ -3+3 & -5+6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$$

Inverse of a 2×2 matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \quad \text{if } ad-bc \neq 0$$

Is $\begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix}$ invertible?

No! since $\det\begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix} = 3\times 2 - 1\times 6 = 6-6 = 0.$

$$\begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix}$$

c_1 c_2

$c_1 = 3c_2$

$1c_1 - 3c_2 = 0$

$$\begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

if $\begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix}$ had an inverse, B say

then $\underbrace{\boxed{B\begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix}}}_{I_2}\begin{pmatrix} 1 \\ -3 \end{pmatrix} = \boxed{B\begin{pmatrix} 0 \\ 0 \end{pmatrix}}$

so $\begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ which is false!

Inverse of a 2×2 matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \quad \text{if } ad-bc \neq 0$$

Find A^{-1} where $A = \begin{pmatrix} 2 & 7 \\ -1 & 3 \end{pmatrix}$ and solve $A\underline{x} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$

$$|A| = 2\times 3 - 7\times(-1)$$
$$= 6 + 7 = 13 \neq 0 \quad \text{so } A^{-1} \text{ does exist}$$

So $A^{-1} = \dfrac{1}{13}\begin{pmatrix} 3 & -7 \\ 1 & 2 \end{pmatrix}$

check

$$\frac{1}{13}\begin{pmatrix} 3 & 7 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 7 \\ -1 & 3 \end{pmatrix}$$
$$= \frac{1}{13}\begin{pmatrix} 6+7 & 0 \\ 0 & 7+6 \end{pmatrix} = \frac{1}{13}\begin{pmatrix} 13 & 0 \\ 0 & 13 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$A\underline{x} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$ $\qquad$ $A^{-1}A = I_2$

$$\underline{x} = \underbrace{A^{-1}A}_{I_2}\underline{x} = A^{-1}(A\underline{x}) = A^{-1}\begin{pmatrix} 1 \\ 6 \end{pmatrix} = \frac{1}{13}\begin{pmatrix} 3 & -7 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 6 \end{pmatrix} = \frac{1}{13}\begin{pmatrix} 3-42 \\ 1+12 \end{pmatrix} = \frac{1}{13}\begin{pmatrix} -39 \\ 13 \end{pmatrix}$$

check $\quad A\begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & 7 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$

$$\underline{x} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$