# Determinant of a 2×2 matrix

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

$\det(A)$ where $A = \begin{pmatrix} 3 & -2 \\ 4 & 0 \end{pmatrix}$

$\det(B)$ where $B = \begin{pmatrix} 2 & 5 \\ 1 & 7 \end{pmatrix}$

$\det\begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix}$

$\begin{vmatrix} 1 & -2 \\ 3 & 5 \end{vmatrix}$

$\begin{vmatrix} 3 & 6 \\ 15 & 30 \end{vmatrix}$

$\det(3A)$

$\det(A^T)$

$\det(AB)$

$\det(A^3)$

$\det(A^{-1})$

# Determinant of a 2×2 matrix

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

$\det(A)$ where $A = \begin{pmatrix} 3 & -2 \\ 4 & 0 \end{pmatrix}$

$$\det(A) = 3 \times 0 - (-2) \times 4$$
$$= 0 + 8$$
$$= 8$$

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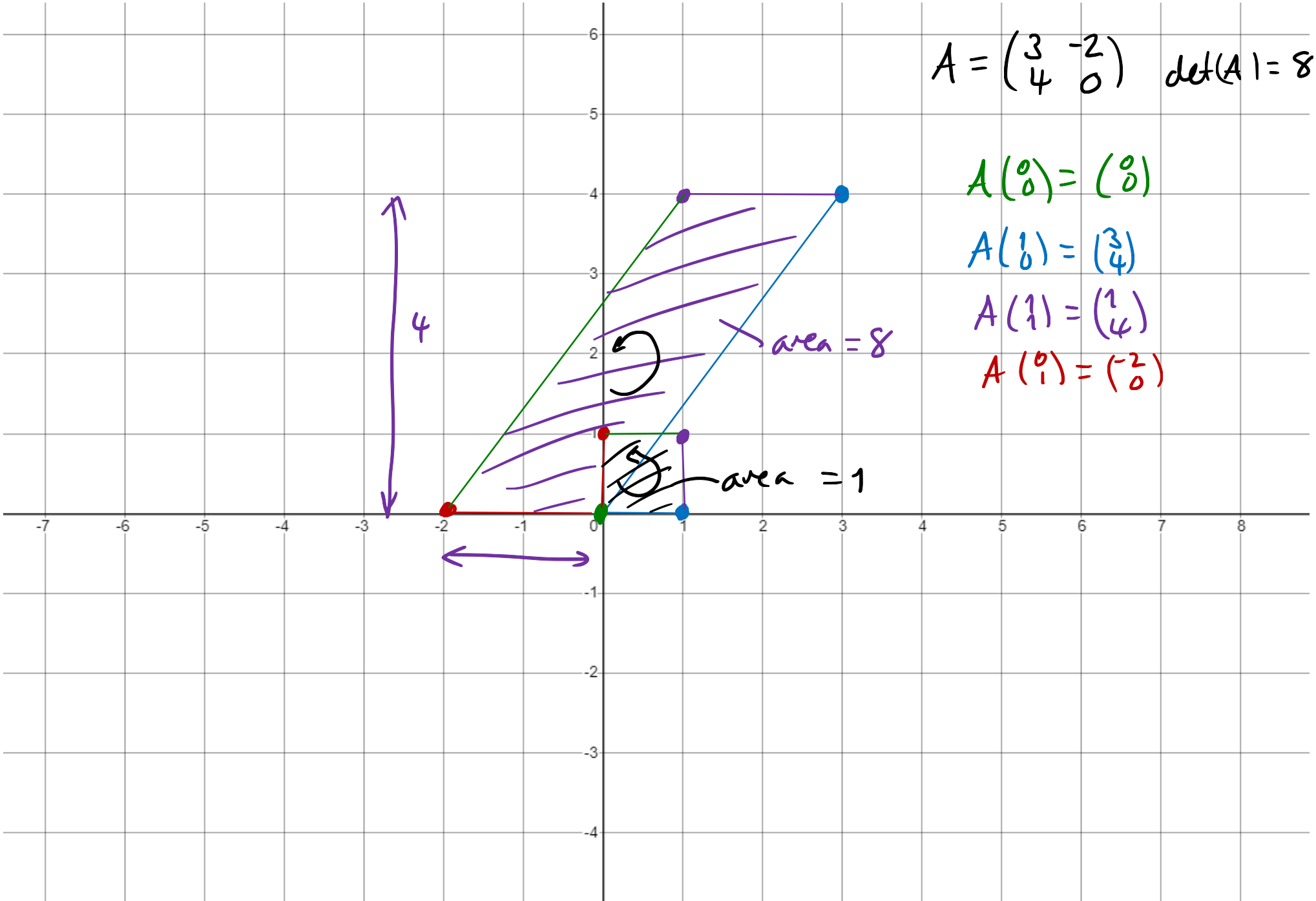
$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$




$$A = \begin{pmatrix} 3 & -2 \\ 4 & 0 \end{pmatrix} \quad \det(A) = 8$$

$$A\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$A\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$A\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$A\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

# Determinant of a 2×2 matrix

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

$\det(B)$ where $B = \begin{pmatrix} 2 & 5 \\ 1 & 7 \end{pmatrix}$ $\quad \det B = 2\times 7 - 1\times 5 = 14 - 5 = \underline{\underline{9}}$

$$\det\begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix} = 4\times 2 - 5\times 3 = 8 - 15 = \underline{\underline{-7}}$$

# Determinant of a 2×2 matrix

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

$$\begin{vmatrix} 1 & -2 \\ 3 & 5 \end{vmatrix} = 1\times 5 - (-2)\times 3 = 5 - (-6) = \underline{\underline{11}}$$

$$\begin{vmatrix} 3 & 6 \\ 15 & 30 \end{vmatrix} = 3\times 30 - 6\times 15$$
$$= 90 - 90$$
$$= 0$$

$$\begin{vmatrix} a & b \\ na & nb \end{vmatrix} = a(nb) - b(na) = abn - abn = 0$$

# Determinant of a 2×2 matrix

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

$B = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$

$\det(B) = \alpha\delta - \beta\gamma$

$$\det(3A) = \begin{vmatrix} 3a & 3b \\ 3c & 3d \end{vmatrix} = (3a)(3d) - (3b)(3c) = 9ad - 9bc = 9(ad-bc)$$
$$= 3^2 \det(A)$$

$$\det(A^T) = \begin{vmatrix} a & c \\ b & d \end{vmatrix} = ad - bc = \det(A)$$

$$AB = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} a\alpha + b\gamma & a\beta + b\delta \\ c\alpha + d\gamma & c\beta + d\delta \end{pmatrix}$$

$$\det(AB) = (a\alpha + b\gamma)(c\beta + d\delta) - (a\beta + b\delta)(c\alpha + d\gamma)$$
$$= ac\alpha\beta + ad\alpha\delta + bc\beta\gamma + bd\gamma\delta$$
$$- ac\alpha\beta - ad\beta\gamma - bc\alpha\delta - bd\gamma\delta$$
$$= (ad - bc)(\alpha\delta - \beta\gamma) = \det A \det(B)$$

$$\det(A^3) = \det(A^2 A) = \det(A^2)\det(A) = \det(AA)\det A = \det A \det A \det A$$
$$= (\det(A))^3$$

$$\det(A^{-1}) = \frac{1}{\det(A)} = (\det A)^{-1}$$

$$A^{-1}A = I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$\det(A^{-1})\det(A) = \det(I_2) = 1$$