Implicit Differentiation

Find $\frac{dy}{dx}$ at $(2, \frac{1}{2})$ when $xy = 1$

Suppose that y is implicitly defined as a function of x by $x^2y^3 + 5xy^2 = 3x + y$

What is $y'(0)$?

Suppose $f(x,y) = x^2y^3 + 5xy^2 - 3x - y$. Verify that $\frac{dy}{dx} = \frac{-f'_x}{f'_y}$.

What is the slope of the tangent to the circle $x^2+y^2=1$ at $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$?

Implicit Differentiation

Find $\frac{dy}{dx}$ at $(2, \frac{1}{2})$ when $xy = 1$

Method ① (Explicit)

$$xy = 1$$
$$y = 1/x$$
$$y = x^{-1}$$
$$\frac{d}{dx}(y) = \frac{d}{dx}(x^{-1})$$
$$\frac{dy}{dx} = -1x^{-2} = \frac{-1}{x^2}$$

so at $(2, \frac{1}{2})$, $\frac{dy}{dx} = \frac{-1}{2^2} = \frac{-1}{4}$

Method ② (Implicit)

$$xy = 1$$
$$\frac{d}{dx}(xy) = \frac{d}{dx}(1)$$
$$1y + x\frac{dy}{dx} = 0$$
$$y + x\frac{dy}{dx} = 0$$
$$x\frac{dy}{dx} = -y$$
$$\frac{dy}{dx} = \frac{-y}{x}$$

so when $(x,y) = (2, \frac{1}{2})$

$$\frac{dy}{dx} = \frac{-(\frac{1}{2})}{2} = \frac{-1}{4}$$

so at $(2, \frac{1}{2})$

$$\frac{1}{2} + 2\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{1}{4}$$

Implicit Differentiation

Suppose that y is implicitly defined as a function of x by $x^2y^3 + 5xy^2 = 3x + y$

what is $y'(0)$?

if $x=0$, $\quad 0 + 0 = 0 + y$, $y = 0$

$$\frac{d}{dx}(x^2y^3 + 5xy^2) = \frac{d}{dx}(3x+y)$$

$$\underbrace{\frac{d}{dx}(x^2y^3)} + \underbrace{\frac{d}{dx}(5xy^2)} = \underbrace{\frac{d}{dx}(3x)}_{3} + \underbrace{\frac{d}{dx}(y)}_{\frac{dy}{dx}}$$

$$= 2xy^3 + x^2\frac{d}{dx}(y^3)$$

$$= 2xy^3 + x^2\, 3y^2\frac{dy}{dx}$$

$$= 5y^2 + 5x\frac{d}{dx}(y^2)$$

$$= 5y^2 + 5x\left(2y\frac{dy}{dx}\right)$$

$$2xy^3 + 3x^2y^2\frac{dy}{dx} + 5y^2 + 10xy\frac{dy}{dx} = 3 + \frac{dy}{dx}$$

$$3x^2y^2\frac{dy}{dx} + 10xy\frac{dy}{dx} - \frac{dy}{dx} = 3 - 2xy^3 - 5y^2$$

$$(3x^2y^2 + 10xy - 1)\frac{dy}{dx} = 3 - 2xy^3 - 5y^2$$

$$\frac{dy}{dx} = \frac{3 - 2xy^3 - 5y^2}{3x^2y^2 + 10xy - 1}$$

So at $x=0$, $y=0$ so $\frac{dy}{dx} = \frac{3-0-0}{0+0-1} = \underline{\underline{-3}}$

Implicit Differentiation

$$\frac{dy}{dx} = \frac{3 - 2xy^3 - 5y^2}{3x^2y^2 + 10xy - 1}$$

Suppose that y is implicitly defined as a function of x by $\boxed{x^2y^3 + 5xy^2 = 3x + y}$ — ⊛

what is $y'(0)$? Suppose $f(x,y) = x^2y^3 + 5xy^2 - 3x - y$. Verify that $\frac{dy}{dx} = \frac{-f_x'}{f_y'}$.

⊛ $\Leftrightarrow$ $f(x,y) = 0$

⊛ is level curve $\mathcal{L}_0$

$$f_x' = 2xy^3 + 5y^2 - 3$$

$$f_y' = 3x^2y^2 + 10xy - 1$$

$$\frac{-f_x'}{f_y'} = \frac{-(2xy^3 + 5y^2 - 3)}{3x^2y^2 + 10xy - 1} = \frac{3 - 2xy^3 - 5y^2}{3x^2y^2 + 10xy - 1} = \frac{dy}{dx}$$

$$f(x,y) = k \iff \frac{dy}{dx} = \frac{-f_x'}{f_y'}$$

Implicit Differentiation

What is the slope of the tangent to the circle $x^2+y^2=1$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$?

$$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(1)$$

$$2x + 2y\,y' = 0$$

$$2y\,y' = -2x$$

$$y' = \frac{-2x}{2y} = \frac{-x}{y}$$

So at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$,

$$y' = -1$$

$$f(x,y) = x^2+y^2 \qquad f(x,y) = 1$$

$$\frac{dy}{dx} = \frac{-f'_x}{f'_y}$$

$$\frac{dy}{dx} = -\frac{2x}{2y} = -\frac{x}{y}$$